Math 1320: Factoring Trinomials

Example 1. Factor to get Prime Factors

$$8x^3 - 8$$

1. The terms have a common factor, both $8x^3$ and 8 are divisible by 8. We can rewrite the polynomial as $8(x^3 - 1)$.
2. Can the factors be factored anymore? Let's repeat the process:
   (a) There are two terms in $x^3 - 1$.
      i. Is it a difference of squares? No, since we have an x^3 term.
      ii. Is it a sum of two cubes? No, since the terms are being subtracted.
      iii. Is it a difference of two cubes? Yes! We can write the polynomial as $8((x)^3 - (1)^3)$. I will apply the formula below and simplify:
      $$\begin{aligned} 8((x)^3 - (1)^3) &= 8[(x-1)((x)^2 + x(1) + (1)^2] \\ &= 8[(x-1)(x^2 + x + 1)] \end{aligned}$$
3. Can the factors be factored anymore? Let's repeat the process:
   (a) We have a factor with three terms: $(x^2 + x + 1)$.
      i. The factor is not a square of a sum or a difference, so let's look at all possible combinations with trial and error. If we look for numbers that have a product of 1 and a sum of 1, there are none. Therefore, we are done!
4. Thus, $8x^3 - 8 \quad = \quad 8(x-1)(x^2 + x + 1)$

Example 2. Factoring Trinomials: Trial and Error

$$x^2 + 5x + 6$$

1. This is a degree 2 polynomial, so we cannot have more than two prime factors: $(\qquad)(\qquad)$
2. Find the first two terms in the parentheses, whose product is x^2: $(x \qquad)(x \qquad)$
3. Find the last two terms in the parentheses, whose product is 6:

   Factors of 6: $1, 6 \mid 2, 3 \mid -1, -6 \mid -2, -3$

4. Try different combinations of these factors. The sum of the outer and inner products must equal $5x$.

Possible Factorizations	**Sum of Outer and Inner Products**
$(x+1)(x+6)$	$x + 6x = 7x$
$(x+2)(x+3)$	$2x + 3x = 5x$
$(x-1)(x-6)$	$-x - 6x = -7x$
$(x-2)(x-3)$	$-2x - 3x = -5x$

5. So, $x^2 + 5x + 6 = (x+2)(x+3)$

$\star$ Note that this method can be more difficult and time consuming when the leading coefficient is not 1. In that case, try the box method or diamond method.

Example 3. Box Method

$$3x^2 - 2x - 5$$

1. Factor out any GCF (in this example the GCF is 1)

2. Multiply the leading coefficient by the constant: $3(-5) = -15$

3. Find two numbers (n, m) such that the product is equal to -15 (the product found in step 1) and the sum is equal to -2 (the coefficient of our x term). In other words:

$$n \cdot m = -15 \qquad \text{and} \qquad n + m = -2$$

   Factors of -15: $-1, 15 \;|\; 1, -15 \;|\; -3, 5 \;|\; 3, -5$

   We know that each of the factors above multiply to be -15, but we need to check which of the factors add up to -2. That's how we chose $n = 3$ and $m = -5$.

4. Create a 2x2 grid and fill in the boxes as follows:
   (a) Upper left: leading term of the polynomial ($3x^2$)
   (b) Upper right: mx term
   (c) Lower left: nx term
   (d) Lower right: constant term of the polynomial (-5)

$3x^2$	$3x$
$-5x$	-5

5. Find the greatest common factor of each row and column. Place them outside the grid.

	x	$+1$
$3x$	$3x^2$	$3x$
-5	$-5x$	-5

6. The factors of the trinomial are made up of the terms outside of the grid.

$$3x^2 - 2x - 5 = (3x - 5)(x + 1)$$

Example 4. Diamond Method

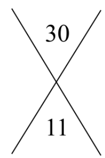
$$2x^2 + 11x + 15$$

1. Factor out any GCF (in this example the GCF is 1)

2. Draw a large diamond (X)

3. Multiply the leading coefficient by the constant: $2(15) = 30$

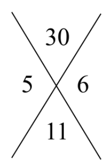
   Place this product in the top quadrant and the coefficient of the x term in the bottom quadrant of the diamond.



4. List all factors of the value found in Step 3:

   Factors of 30: $1, 30 \ | \ 2, 15 \ | \ -3, 10 \ | \ 5, 6 \ | \ -1, -30 \ | \ -2, -15 \ | \ -3, -10 \ | -5, -6$

5. Find the two factors (call them n, m) in step 4, whose sum is equal to the value at the bottom of the diamond (in this example, 11). Place these values in the left and right quadrants of the diamond.



6. Rewrite the original trinomial replacing the x term with the sum $nx + mx$:

$$2x^2 + 5x + 6x + 15$$

7. Factor by grouping:

$$\begin{aligned} 2x^2 + 5x + 6x + 15 \quad &\to (2x^2 + 5x) + (6x + 15) \\ &\to x(2x + 5) + 3(2x + 5) \\ &\to (x + 3)(2x + 5) \end{aligned}$$

Practice Problems

Try factoring these polynomials using the methods above. Answers are provided.

1. $9x^3 + 3x^2 + 15x + 5 \ = \ (3x^2 + 5)(3x + 1)$

2. $3x^2 - 13x - 10 \ = \ (3x + 2)(x - 5)$

3. $2x^2 + 11x - 6 \ = \ (x + 6)(2x - 1)$